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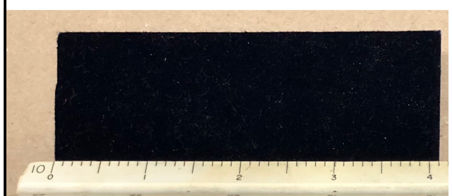
$$e_{R_T} = \sqrt{\frac{V_{out}^{\ 2}}{\left(V_{in} - V_{out}\right)^2} e_{R_1}^{\ 2} + \frac{R_1^{\ 2} V_{out}^{\ 2}}{\left(V_{in} - V_{out}\right)^4} e_{V_{in}}^{\ 2} + \frac{R_1^{\ 2} V_{in}^{\ 2}}{\left(V_{in} - V_{out}\right)^4} e_{V_{out}}^{\ 2}}.$$



SIGNIFICANCE LEVEL FOR TWO-TAILED TEST						
df	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610

Lecture 2A – Introduction to Error Analysis

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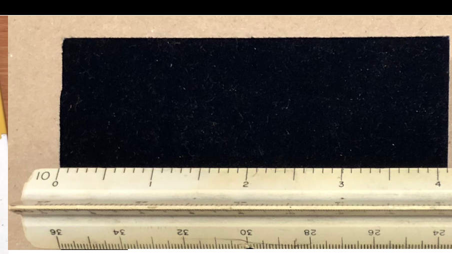
Overview Measurements



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Reporting Uncertainty

$$m = 1.03 \pm 0.03 \text{ kg (95\% confidence)}$$

$$v = 2.36 \pm 0.04 \text{ m/s (95\% confidence)}$$

$$mv = 2.43 \pm 0.08 \text{ kg} \cdot \text{m/s (95\% confidence)}$$

$$\frac{1}{2}mv^2 = 2.87 \pm 0.13 \text{ J (95\% confidence)}$$

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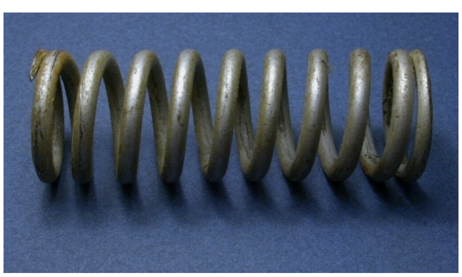
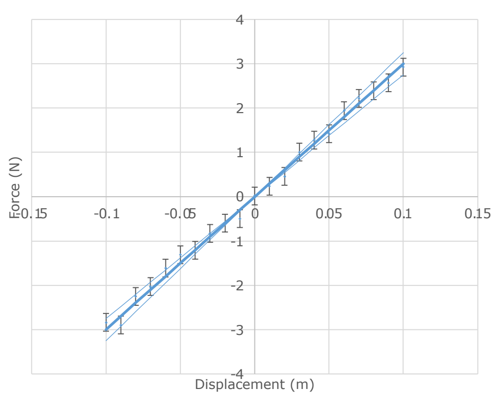
Reporting Uncertainty of Functions

$$F_s = k(x_n - x_f)$$

F_s

x_f x_n





$$F_s = 30.00 \pm 2.50 \text{(95\% conf.)N/m} \left(x_n - x_f\right)$$

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E80 Experimental Engineering — True Mean & Standard Deviation

Infinite Precision Exact Measurement

Full Population Measurement

$$\text{True Mean or Population Mean} \equiv \mu$$

$$\text{True Standard Deviation} \equiv \sigma$$

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E80 Experimental Engineering — Sample Mean & Residuals

The set of measurements

$$\{x_1, x_2, \cdots x_N\}$$

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \approx \mu$$

How do we estimate $\mu - \bar{x}$?

Can we calculate the error

$$\varepsilon_i = \mu - x_i \text{ ?}$$

The set of errors $\{\varepsilon_1, \varepsilon_2, \cdots \varepsilon_N\}$

Calculate the residual $e_i = \bar{x} - x_i$

The set of residuals $\{e_1, e_2, \cdots e_N\}$

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Sample Variance and Standard Dev.

Sample variance

$$S^2 = \frac{\sum_{i=1}^{N} e_i^2}{N-1}$$

$\bar{x}$ is calculated from the x_i.
μ is not

True variance

$$\sigma^2 = \frac{\sum_{i=1}^{N} \varepsilon_i^2}{N}$$

Number of independent values in calculation

Sample standard deviation: $S = \sqrt{S^2}$

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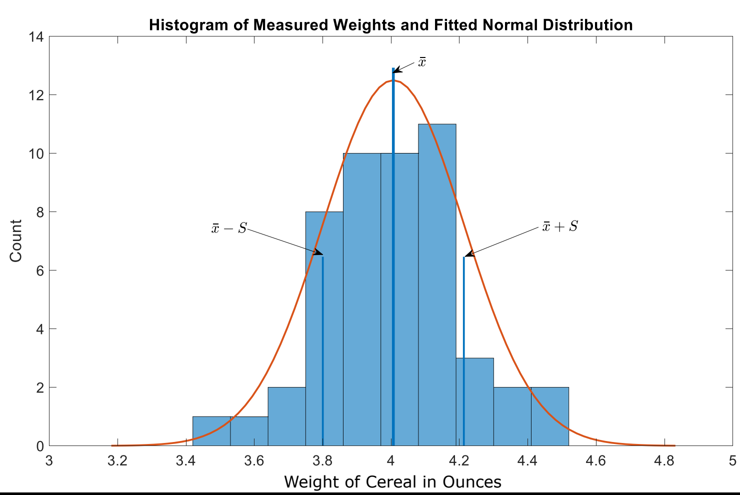
Are We There Yet?

Can we estimate $\mu - \bar{x}$ yet?

No, but we can estimate the spread of our data from S.



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E80 Experimental Engineering — Standard Error

$$\text{St. Err.} = \sigma_\mu = \frac{\sigma}{\sqrt{N}} \approx \frac{S}{\sqrt{N}}$$

$$\text{Est. St. Err.} = S_{\bar{x}} = \frac{S}{\sqrt{N}}$$

For enough points $\mu = \bar{x} \pm S_{\bar{x}}$ (68% conf.)

For example, $y = 42.000 \pm 0.007$(68% conf.)

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E80 Experimental Engineering — Student's t

By Sami Keinänen - www.flickr.com, CC BY-SA 2.0, https://commons.wikimedia.org/w/index.php?curid=802514

By User Wujaszek on pl.wikipedia - scanned from Gosset's obituary in Annals of Eugenics, Public Domain, https://commons.wikimedia.org/w/index.php?curid=1173662

Calculate $\bar{x}$ and S.

Calculate $S_{\bar{x}}$.

Choose a confidence level,
For example, 95% or $p = 0.05$.

Find t given p and $df = N - 1$.

Then $\lambda = tS_{\bar{x}}$
and $\mu = \bar{x} \pm \lambda(1 - p \text{ conf.})$

For example, $\bar{x} = 42.000 \pm 0.067$(95% conf.)

Go to https://en.wikipedia.org/wiki/Begging_the_question

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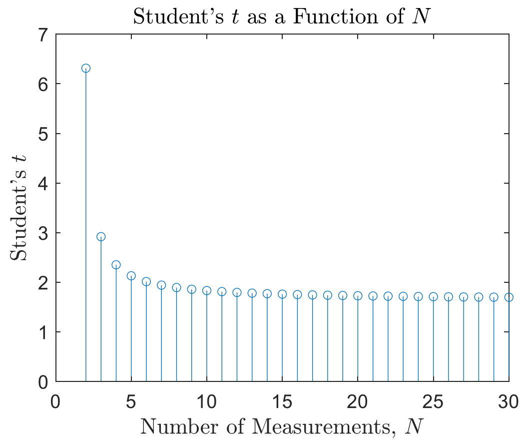
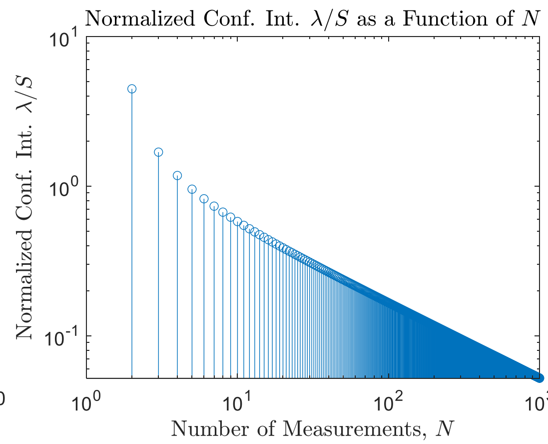
What Does It Mean?

$$\lambda = tS_{\bar{x}} = \frac{tS}{\sqrt{N}}$$





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Insert LabVIEW Demo Here

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Takaways

1. Make at least three measurements.
2. Calculate the confidence interval from the estimated standard error and the Student's t value.
3. Report your results with the confidence interval and the confidence level, e.g., 42.000±0.067(95% conf.).

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