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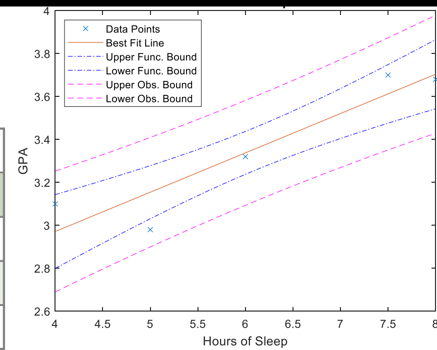
$$SSE = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left[y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i \right) \right]^2$$

SIGNIFICANCE LEVEL FOR TWO-TAILED TEST						
df	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610



Lecture 2E – Linear Regression

1

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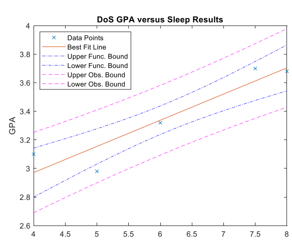
Experimental Engineering

Overview of Linear Regression

- Take data.
- Assume linear.
- Perform regression.
- Report slope and intercept.
- Plot data, line, uncertainties.

$$\hat{\beta}_1 = 0.183 \pm 0.067 (80\% \text{ conf.})$$

$$\hat{\beta}_0 = 2.238 \pm 0.418 (80\% \text{ conf.})$$



Partial list of techniques:

- Least Squares
- Least Absolute Deviation (LAD)
- Median-Median
- Least Median of Squares (LMS)

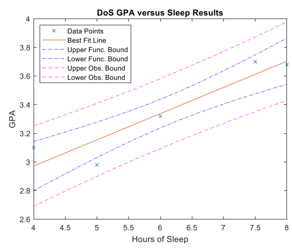
2

E80 Experimental Engineering — Overview of Linear Regression

- Take data.
- Assume linear.
- Perform regression.
- Report slope and intercept.
- Plot data, line, uncertainties.

$$\hat{\beta}_1 = 0.183 \pm 0.067 (80\% \text{ conf.})$$

$$\hat{\beta}_0 = 2.238 \pm 0.418 (80\% \text{ conf.})$$



Partial list of techniques:

- **Least Squares**
- Least Absolute Deviation (LAD)
- Median-Median
- Least Median of Squares (LMS)

3

E80 Experimental Engineering — Measurements and Residuals

The set of measurements

$$\{(x_1, y_1), (x_2, y_2), \cdots, (x_N, y_N)\}$$

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

Can we calculate the error?

$$\varepsilon_i = y_i - \left(\beta_0 + \beta_1 x_i\right)$$

The set of errors $\{\varepsilon_1, \varepsilon_2, \cdots \varepsilon_N\}$

Calculate the residual

$$e_i = y_i - \hat{y}_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)$$

The set of residuals

$$\{e_1, e_2, \cdots e_N\}$$

The Sum of Squared rEsiduals, SSE

$$\sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left(y_i - \hat{y}_i\right)^2 = \sum_{i=1}^{N} \left[y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right)\right]^2$$

4

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Best-Fit Slope & Intercept

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Root Mean Squared rEsidual ($RMSE$)

$$RMSE = S_e = \sqrt{\frac{SSE}{N-2}} = \sqrt{\frac{\sum_{i=1}^{N} e_i^2}{N-2}}$$

Lose two degrees of freedom by calculating $\hat{\beta}_0, \hat{\beta}_1$

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Standard Errors & C.I.'s

Standard Errors

$$S_{\beta_0} = S_e \sqrt{\frac{1}{N} + \frac{\bar{x}^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}}$$

$$S_{\beta_1} = S_e \sqrt{\frac{1}{\sum_{i=1}^{N}(x_i - \bar{x})^2}}$$

Confidence Intervals

$$\lambda_{\beta_0} = t S_{\beta_0}$$

Remember: $df = N - 2$

$$\lambda_{\beta_1} = t S_{\beta_1}$$

6

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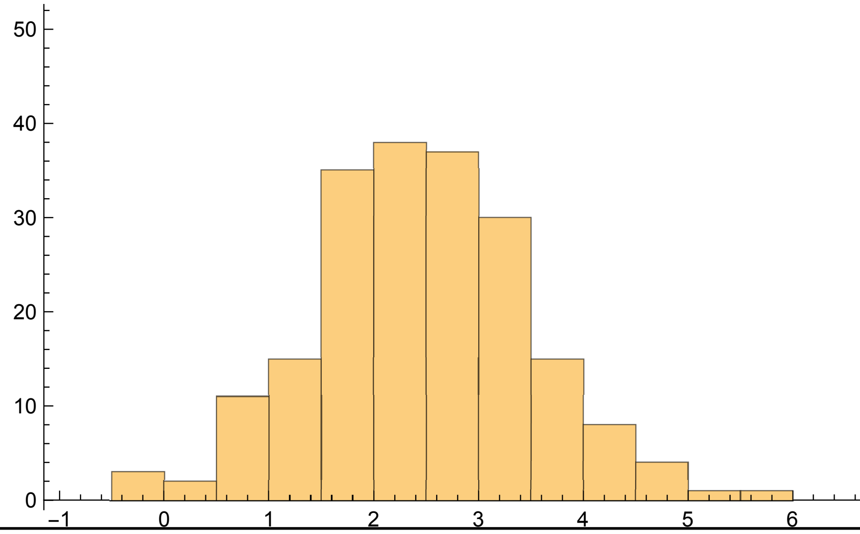
Histograms to Lines



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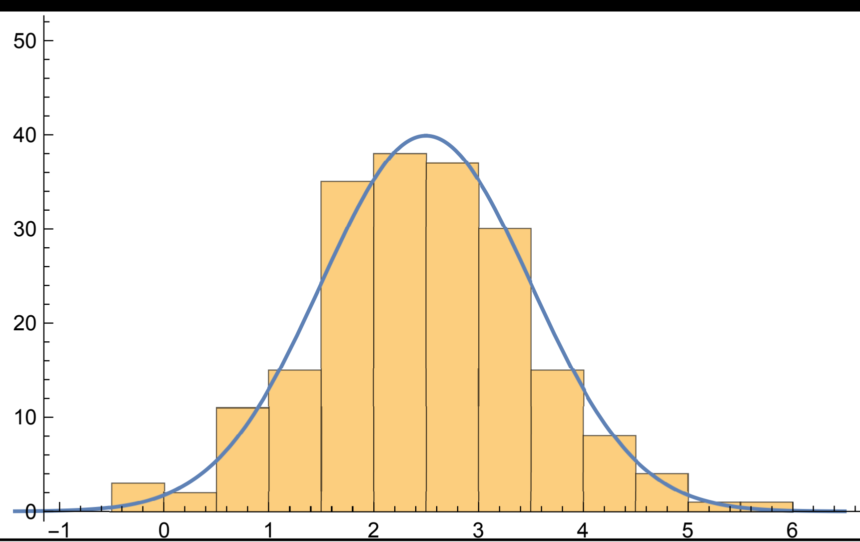
Histograms to Lines



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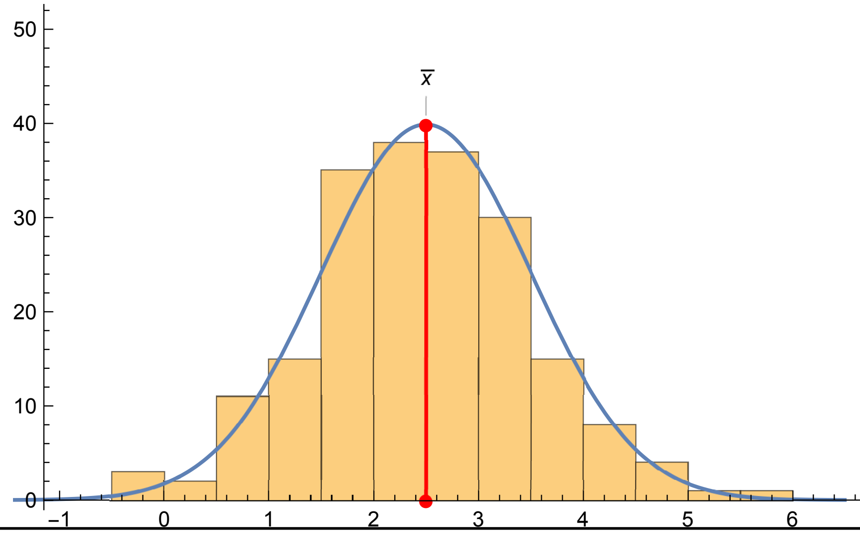
Histograms to Lines



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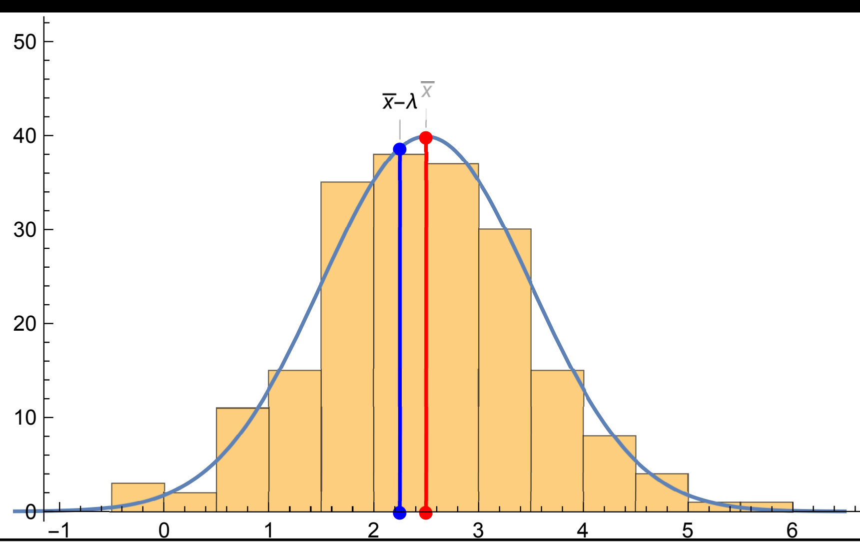
Histograms to Lines



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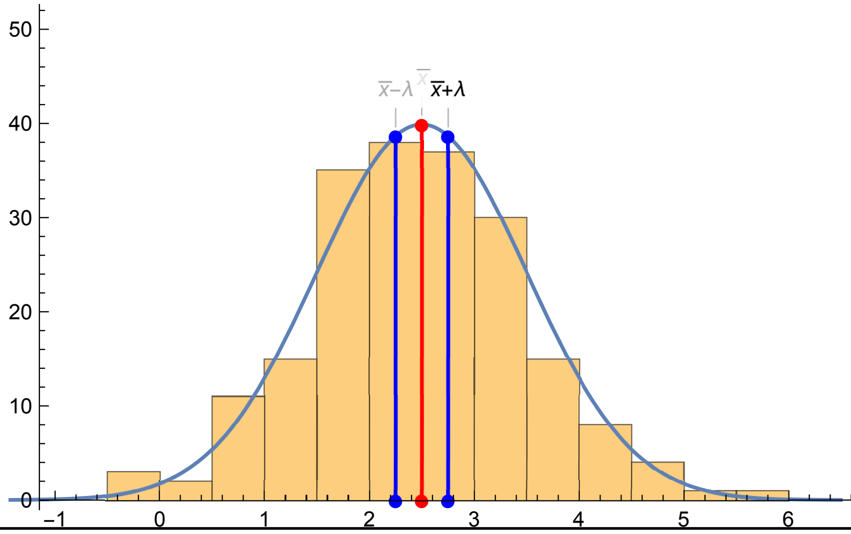
Histograms to Lines



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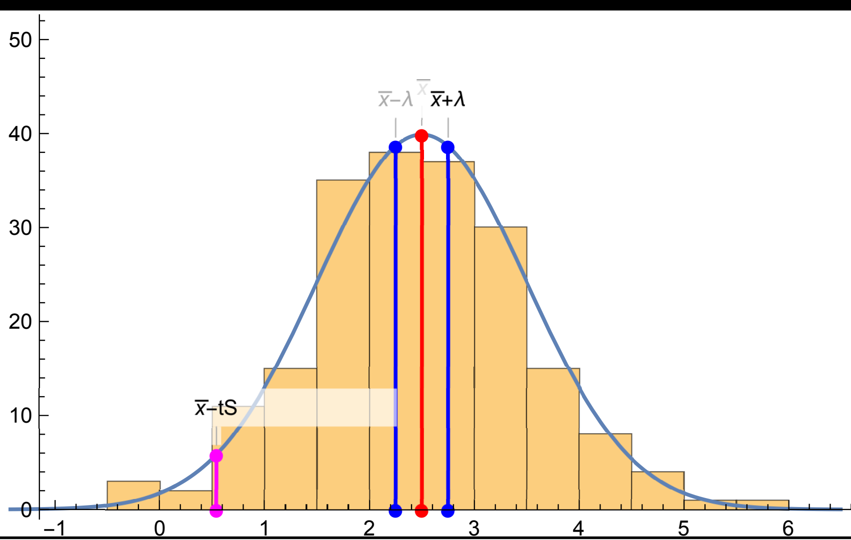
Histograms to Lines



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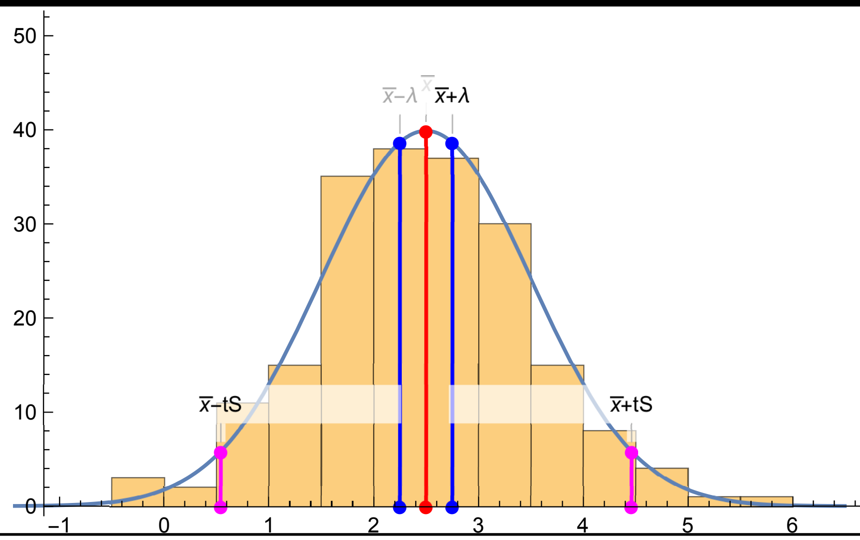
Histograms to Lines



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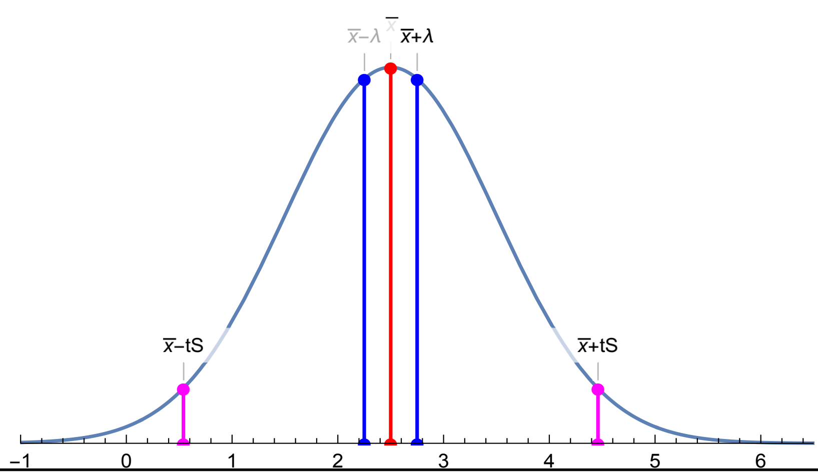
Histograms to Lines



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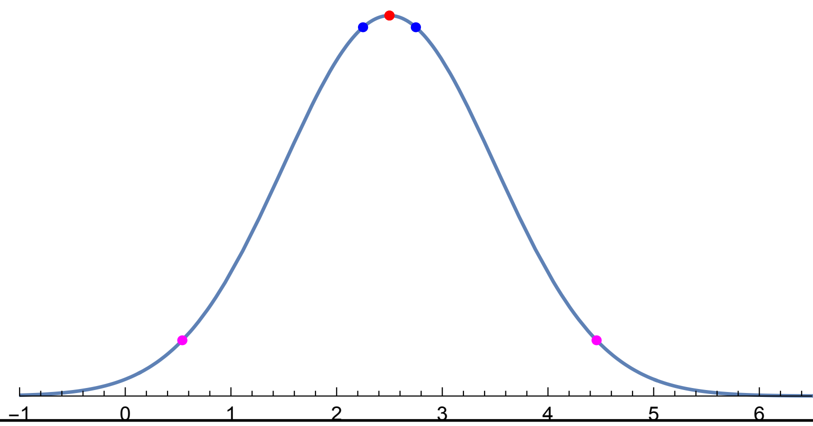
Experimental Engineering

Histograms to Lines



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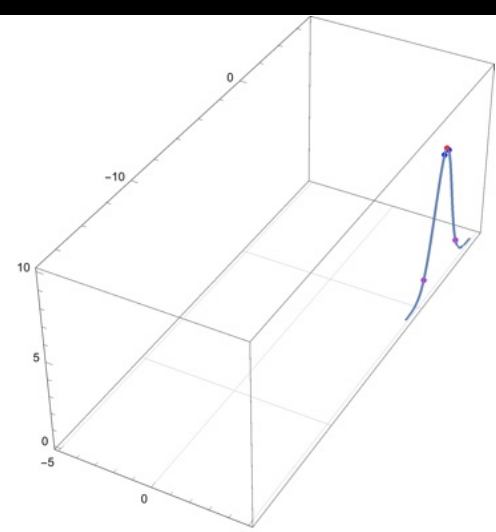
Experimental Engineering

Histograms to Lines



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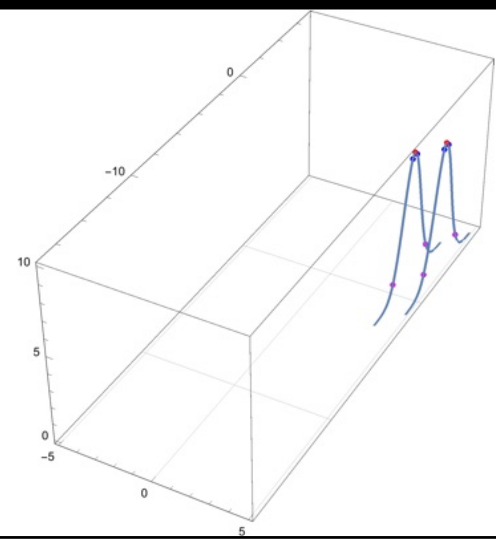
Experimental Engineering

Histograms to Lines



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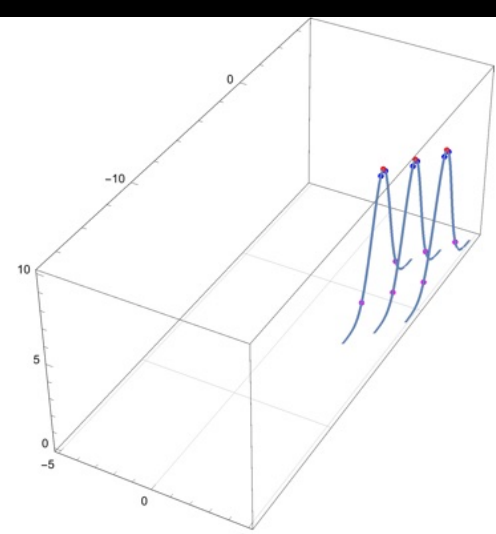
Experimental Engineering

Histograms to Lines



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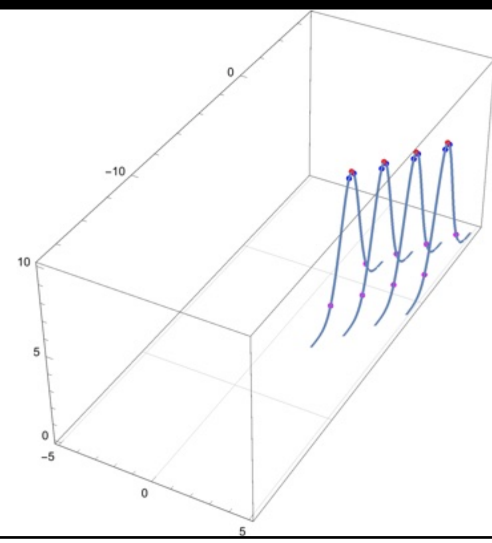
Histograms to Lines



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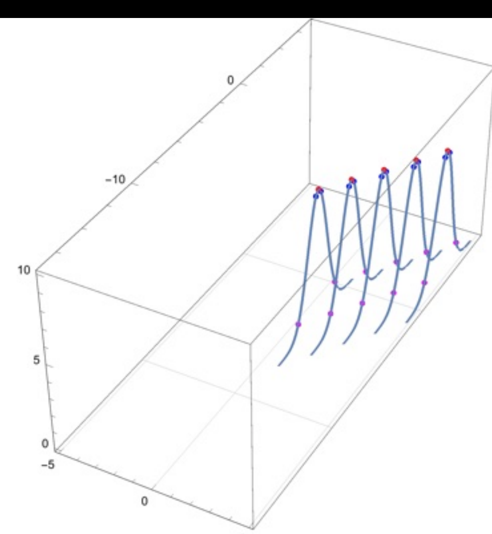
Histograms to Lines



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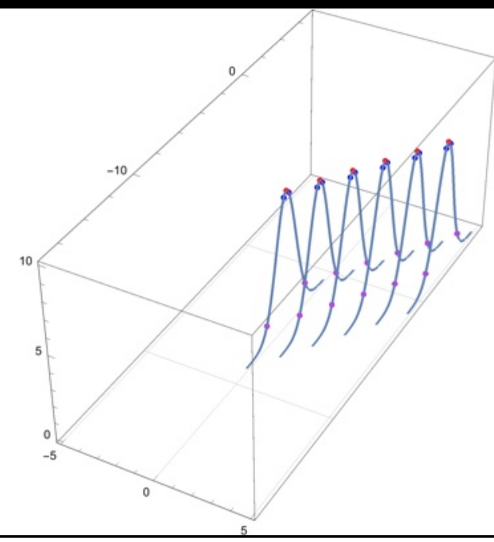
Histograms to Lines



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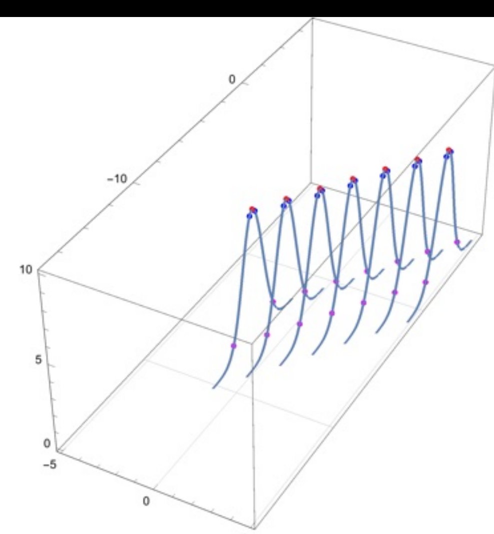
Histograms to Lines



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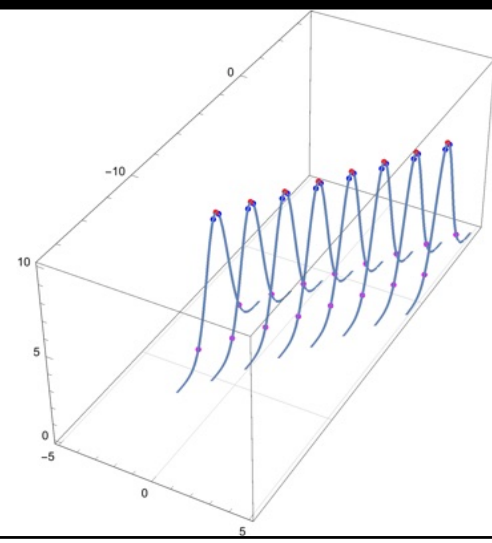
Experimental Engineering

Histograms to Lines



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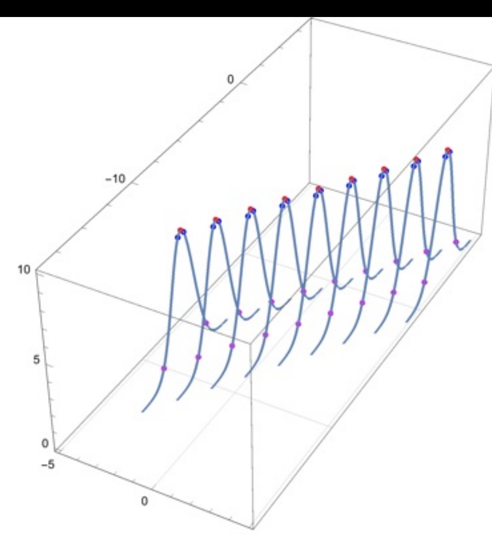
Experimental Engineering

Histograms to Lines



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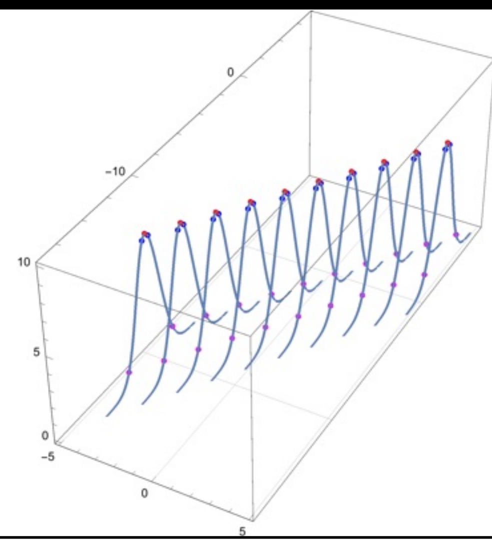
Experimental Engineering

Histograms to Lines



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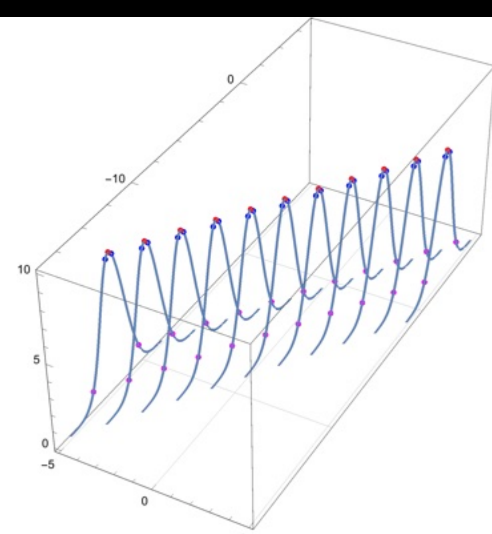
Experimental Engineering

Histograms to Lines



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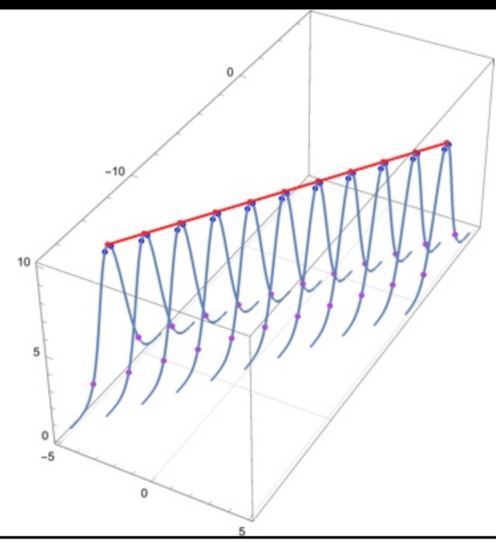
E80 Experimental Engineering — Histograms to Lines



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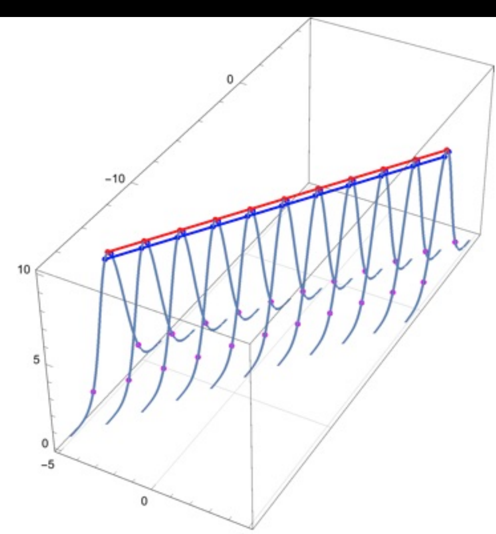
E80 Experimental Engineering — Histograms to Lines



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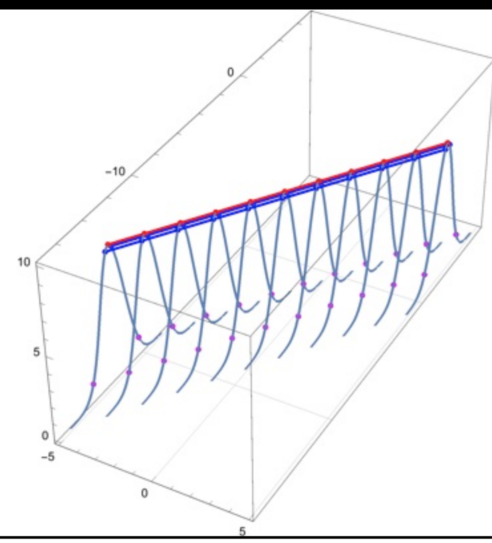
Histograms to Lines



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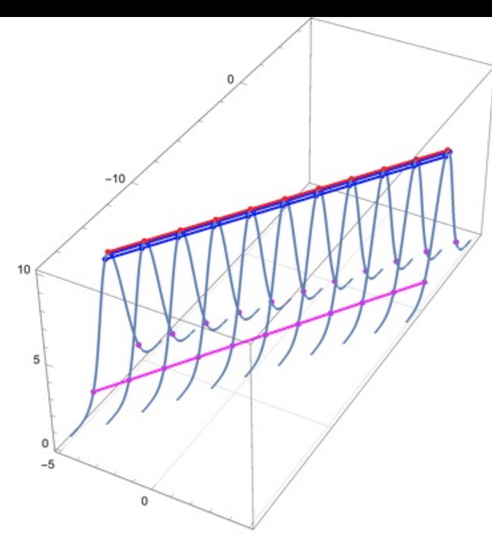
Histograms to Lines



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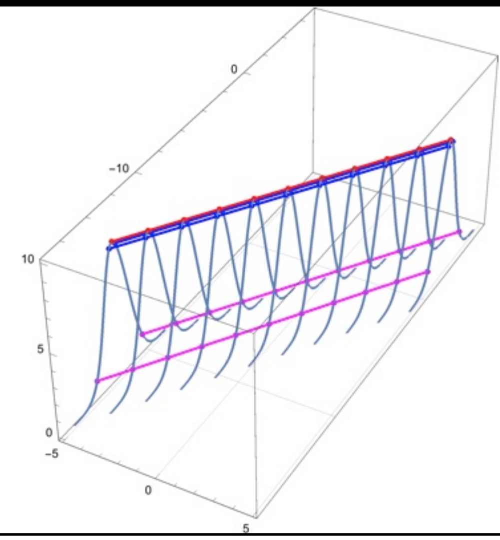
Histograms to Lines



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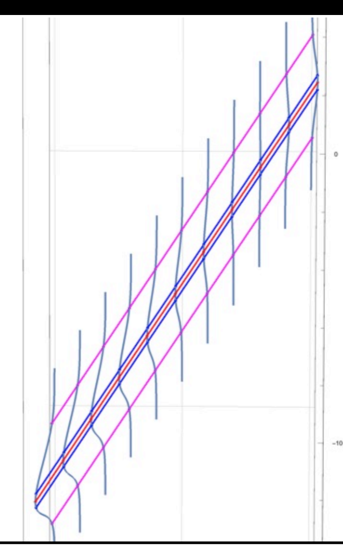
Histograms to Lines



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Confidence Bounds

Confidence bound on line or mean value $\hat{y}$ at x (functional bound)

$$S_{\hat{y}} = S_e \sqrt{\frac{1}{N} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}} \qquad \lambda_{\hat{y}} = tS_{\hat{y}}$$

$$y_{ucb\hat{y}} = \hat{\beta}_0 + \hat{\beta}_1 x + \lambda_{\hat{y}}$$

$$y_{lcb\hat{y}} = \hat{\beta}_0 + \hat{\beta}_1 x - \lambda_{\hat{y}}$$

Confidence bound on data (observational bound)

$$S_y = S_e \sqrt{1 + \frac{1}{N} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}} \qquad \lambda_y = tS_y$$

$$y_{ucby} = \hat{\beta}_0 + \hat{\beta}_1 x + \lambda_y$$

$$y_{lcby} = \hat{\beta}_0 + \hat{\beta}_1 x - \lambda_y$$

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Example

	x_i (Hours of Sleep)	y_i (GPA)
	7.5	3.70
	4.0	3.10
	6.0	3.32
	5.0	2.98
	8.0	3.68

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} = 0.183 \qquad S_{\beta_1} = S_e \sqrt{\frac{1}{\sum_{i=1}^{N}(x_i - \bar{x})^2}} = 0.0407$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 2.238 \qquad S_{\beta_0} = S_e \sqrt{\frac{1}{N} + \frac{\bar{x}^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}} = 0.255$$

$$S_e = \sqrt{\frac{\sum_{i=1}^{N} e_i^2}{N-2}} = 0.136 \qquad t(df = 3, 80\% \text{ conf.}) = 1.638$$

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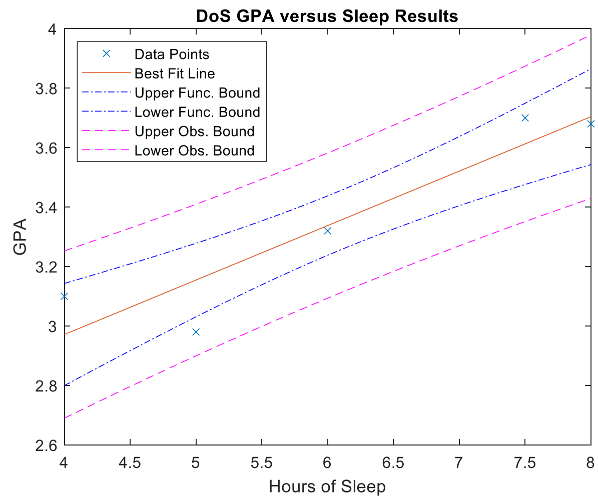
Example (cont.)

	x_i (Hours of Sleep)	y_i (GPA)
	7.5	3.70
	4.0	3.10
	6.0	3.32
	5.0	2.98
	8.0	3.68
mean	**6.1**	**3.36**

$$\hat{\beta}_1 = 0.183 \pm 0.067 (80\% \text{ conf.})$$

$$\hat{\beta}_0 = 2.238 \pm 0.418 (80\% \text{ conf.})$$



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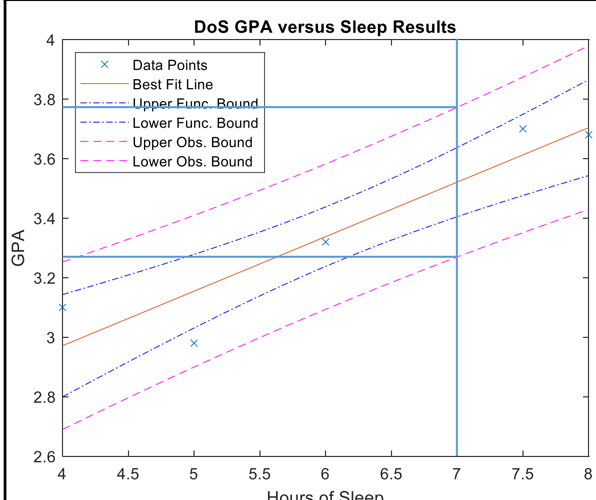
Example (cont.)



$$y = 2.238 + 0.183 \bullet 7 = 3.52$$

$$S_y = S_e \sqrt{1 + \frac{1}{N} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2}} = 0.154$$

$$\lambda_y = t S_y = 0.254$$

$$y(7) = 3.52 \pm 0.254 = \begin{bmatrix} 3.27 & 3.77 \end{bmatrix}$$

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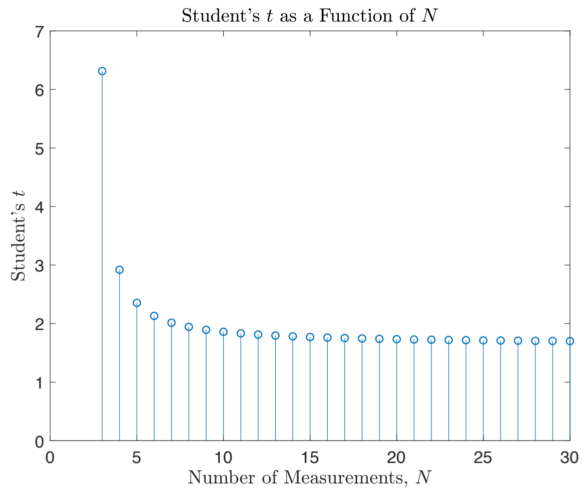
What Does It Mean?

$$\lambda_{\hat{y}} = t S_{\hat{y}}$$



$$S_{\hat{y}} = S_e \sqrt{\frac{1}{N} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}}$$

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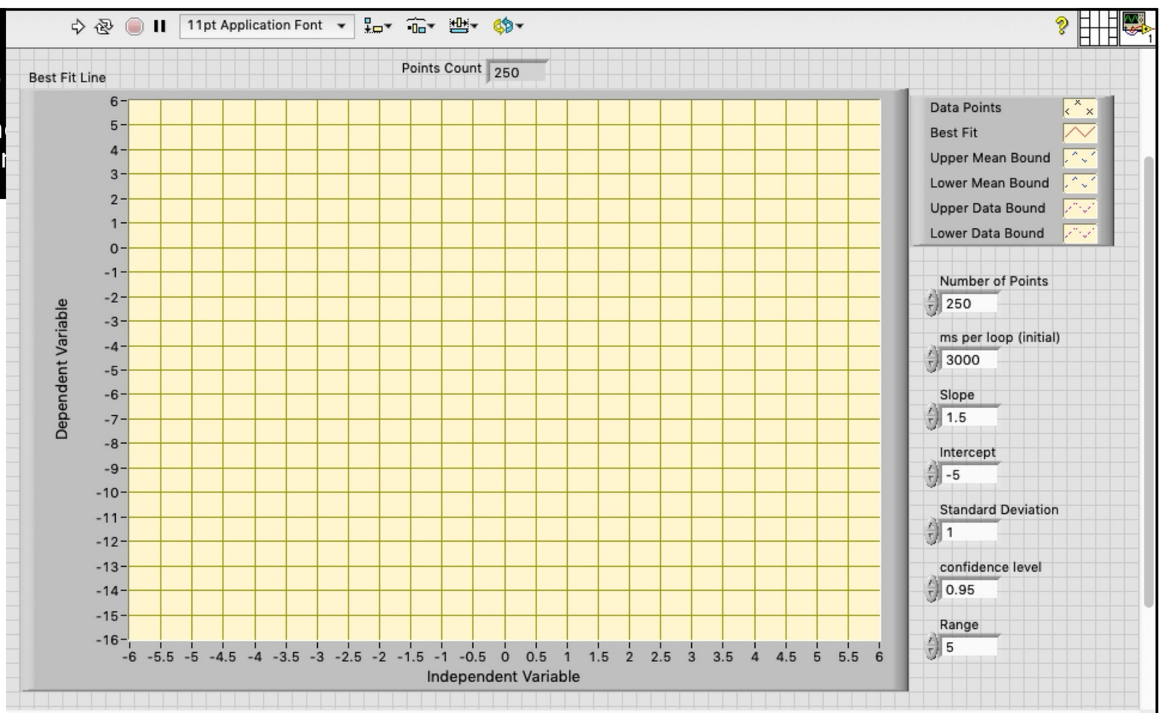
Engineer



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Experimental Engineering

Takaways

1. For a line, make at least four measurements.
2. DO NOT calculate these things by hand. Use a software package.
3. Report your results with the confidence interval and the confidence level, e.g., $\hat{\beta}_1 = 0.183 \pm 0.067 (80\% \text{ conf.})$ $\hat{\beta}_0 = 2.238 \pm 0.418 (80\% \text{ conf.})$
4. Plot data points, best-fit curve, function confidence bound, data confidence bound.

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